Indian Statistical Institute, Bangalore Centre.
Mid-Semester Exam : Probability 1

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Date : November 27th , 2019.

Max. points : 10.   **Time Limit : 1.5 hours.**

Answer any two questions only. All questions carry 5 points.

Give necessary justifications and explanations for all your arguments. If you are citing results from the class or assignments, mention it clearly.
Always define the underlying probability spaces, events and random variables clearly before computing anything !

1. 10 balls labelled $1, 2, \ldots, 10$ are thrown uniformly at random into 4 labelled bins (i.e., balls are thrown one by one independently into bins chosen uniformly at random). In each of the following cases, prove or disprove that A and B are independent and also compute $\mathbb{P}(B|A)$ and $\mathbb{P}(B)$.

   (a) Let A be the event that the first bin contains an even numbered ball. Let B be the event that the second bin contains an odd numbered ball.

   (b) Let A be the event that the third bin contains a ball with label at most 4 and B be the event that the fourth bin contains a ball of label at least 7.

2. A man possesses five coins, two of which are double-headed, one is double- tailed, and two are normal. He shuts his eyes, picks a coin at random, and tosses it. He opens his eyes and sees that the coin is showing heads; (a) Find the probability that the lower face is a head? and (b) He shuts his eyes again, and tosses the coin again. Find the probability that the lower face is a head?

3. Let N be the number of empty poles when r flags of different colours are displayed uniformly at random on n poles arranged in a row (here $r, n \in \mathbb{N}$). Assuming that there is no limitation on the number of flags on each pole, compute the pmf and expectation of N.

4. A pack of cards consists of n distinct cards (label them $1, \ldots, n$) being repeated s many times. Suppose a random sample of size r ($r \geq n$ is drawn from the pack of cards without replacement what is the probability that each of the labelled cards are present in the sample ?